SIMPLIFIED VARIANT OF EASILY INTEGRATABLE STRESS-STRAIN RELATIONSHIP FOR CONCRETE

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Abstract
The authors of this paper offer to use formula of simplified variant of easily integratable stress-strain relationship instead of the formula of regulation STR 2.05.05:2005 for calculation of normal concrete stresses of reinforced concrete members made of heavy and fine-grained concrete strength classes C25/30–C90/105. It is reasonable to do this in cases when there is a need to integrate the non-linear stress-strain diagram.
KEY WORDS: *heavy and fine-grained concretes, normal stresses, non-linear diagram.*

Anotacija
Sunkiojo ir smulkiagrūdžio C25/30–C90/105 stiprio klasių betono normaliniams įtempiams gelžbetoninių elementų betone apskaičiuoti vietoje reglamento STR 2.05.05:2005 formulės galima naudoti straipsnio autorių siūlomą lengvai integruojamos betono įtempių-deformacijų priklausomybės supaprastinto varianto formulę. Tai daryti tikslinga, kai tenka integruoti kreivinę betono įtempių diagramą.
PAGRINDINIAI ŽODŽIAI: *sunkusis ir smulkiagrūdis betonai, normaliniai įtempiai, kreivinės diagramos.*

Introduction

During the development and research of structures it is often necessary to calculate the stress-strain state of reinforced concrete members for sections perpendicular to the axis (normal sections). These calculations must be performed at various stages of loading, i. e.:

- *before cracking* (calculation of the stresses for prestressed concrete, calculation of the losses of prestress for reinforced concrete members);
- *at cracking stage* (determination of cracking moment);
- for members with cracks *at the service stage* (forecast of crack widths, curvature and stiffness of members);
- *at failure stage* (determination of members' bearing strength).

Simple but *very conditional stress-strain diagrams* are used for the calculation of parameters of the aforesaid states in regulation [1] and Eurocode 2 [2]:

- *triangular* diagrams of elastic materials are taken to calculate the concrete stresses before cracking;
- the cracking moments are calculated according to a very inaccurate model: in the tensile zone – *a rectangular* diagram, in the compressive zone – *a non-linear* diagram which is not clearly defined;
- the empirical formulae to calculate the crack widths, the curvature (deformation) and the stiffness and equations for members' strength were derived having assumed *a rectangular* concrete stresses diagram in compressive zone of cross-section; *the stresses* of the tensioned concrete above the crack *are absolutely disregarded* in this case.

In the absence of a uniform calculation method and in case roughly distorted calculation models are assumed:

- numerous correction coefficients need to be used;
- the empiric formulae of these correction coefficients sometimes are very bulky and complicated and formulation of such formulae is costly;
- there is no possibility to calculate the height of the cracks and the height of the tensile zone of the concrete above them;

- it is impossible to define the stress-strain state of reinforced concrete members according to the measured parameters for the stages before cracking and before failure;
- it is impossible to define the stress-strain state in plastic hinges of the continuous beams, i.e. when the strength of the member decreases due to the fact that the stresses of the reinforcement exceed the yield limit or/and the stresses of the compressive zone of concrete exceed the strength limit of concrete;
- there is no method to define the stress-strain state of reinforced concrete members according to the measured parameters of the cracks (the width and the height of the cracks and the distance between them) in stages when their loads exceed the service loads.

It is possible to solve many of these problems using a more realistic non-linear diagram of stress-strain relationship for concretes named in this paper. Such a non-linear diagram is presented in the regulation [1] and other documents [2, 3]. However, its use is aggravated by the integral calculation problems.

The purpose of this work is to describe the most general non-linear diagram of relationship between stresses and strains of concrete presented in the standard document [1] so that it would be easy to integrate and so that it would be applicable to concrete strength classes C25/30–90/105.

The substance and results of the work

The present paper deals with the calculation of normal concrete stresses in the reinforced concrete members made of heavy and fine-grained concretes strength classes C12/15…C90/105.

The key symbols are the same as in standard documents [1, 3]:

σ_c – normal compressive stresses of concrete;

f_{cm} – mean value of concrete cylinder compressive strength;

f_{ck} – characteristic compressive cylinder strength of concrete at 28 days;

E_{cm} – secant modulus of elasticity of concrete;

ε_c – compressive strain of concrete;

ε_{c1} – compressive strain of concrete corresponding to f_{cm};

ε_{cu1} – ultimate compressive strain of concrete.

SI units used in the paper:

of stress strength – *MPa* (*MN/m²=N/mm²*);
of elastic modulus – *GPa* (*GN/m²*);
of strain – per mile (*‰*).

Fig.1 shows non-linear relationship between stresses and strains ($\sigma_c - \varepsilon_c$) of concrete specified in the concrete and reinforced concrete design regulation [1] adapted for the Eurocode 2 area [2, 3].

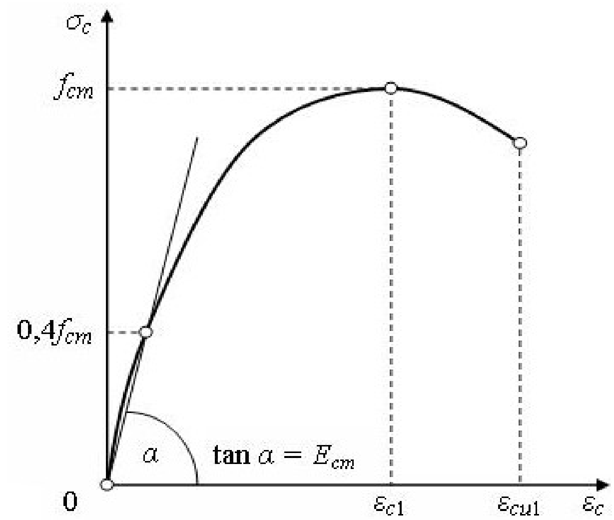

Fig. 1. Stress-strain relationship for concrete (formula (1))

This $\sigma_c - \varepsilon_c$ relationship is described by the expression:

$$\frac{\sigma_c}{f_{cm}} = \frac{k\eta - \eta^2}{1+(k-2)\eta}; \tag{1}$$

here

$$\eta = \varepsilon_c / \varepsilon_{c1},$$

$$k = \frac{1{,}1 E_{cm} \cdot \varepsilon_{c1}}{f_{cm}}.$$

In the expression (1) and farther the values of ε_{c1} and ε_{cu1} are considered to be positive. If $E_c = 1{,}1E_{cm}$ and $\nu_{c1} = f_{cm} / E_c \varepsilon_{c1}$, then $k = 1/\nu_{c1}$ and

$$\sigma_c = \frac{\eta/\nu_{c1} - \eta^2}{1+(1/\nu_{c1} - 2)\eta} f_{cm}. \tag{1a}$$

Formula (1) is simple, but unhandy when there is a need to integrate the non-linear stress-strain diagrams.

In the proposed method [4–7] a non-linear $\sigma - \varepsilon$ diagram is used (see Fig. 2) for the calculation of the parameters of stresses and strains of reinforced concrete members for normal sections. It is described by the following formula:

$$\sigma_c = E_c \varepsilon_c (1 + c_1\eta + c_2\eta^2 + c_3\eta^3 + c_4\eta^4) = \nu_c E_c \varepsilon_c = \nu_c \sigma_C. \tag{2}$$

Analysis of the formula (1) showed that in the interval of concrete strains ε_c = 0… ε_{cu1} the formula (2) can be simplified, i. e. one can take $c_3 = c_4 = 0$ for concretes strength classes C25/30…C90/105.

Then

$$\sigma_c = E_c \varepsilon_c (1 + c_1\eta + c_2\eta^2) = \nu_c E_c \varepsilon_c = \nu_c \sigma_C; \tag{2a}$$

here

$$\nu_c = 1 + c_1\eta + c_2\eta^2 = 1 + (3\nu_{c1} - 2)\eta + (1 - 2\nu_{c1})\eta^2, \tag{3}$$

$$c_1 = 3\nu_{c1} - 2, \quad c_2 = 1 - 2\nu_{c1};$$

$$E_c = \tan\beta \quad \text{(Fig. 2)};$$

$$\sigma_{C1} = E_c \varepsilon_{c1}; \qquad \sigma_{c1} = f_{cm}; \qquad \nu_{c1} = \sigma_{c1} / \sigma_{C1}.$$

Here values of β and indexes *c* and C are shown in Fig. 2.

Programmes are made for the calculation of the values of stresses σ_c by formulae (1a) and (2a). It is necessary to inscribe only one value of parameter f_{ck} in these programmes during the calculation of σ_c.

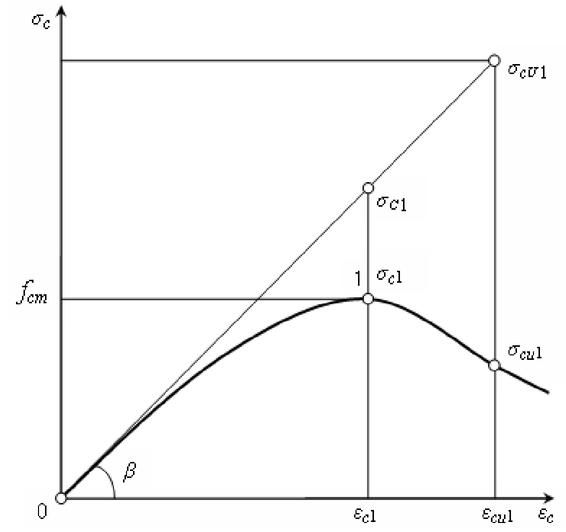

Fig. 2. Stress-strain relationship for concrete (formula (2a))

The order of calculation:

1) values of the following parameters are calculated from formulae presented in regulation [1]:

$$f_{cm} = f_{ck} + 8; \quad (4)$$

$$E_{cm} = 22(f_{cm}/10)^{0,3}; \quad (5)$$

$$E_c = 1{,}1E_{cm}; \quad (6)$$

$$\varepsilon_{c1} = 0{,}7 f_{cm}^{0,31}; \quad (7)$$

$$\varepsilon_{cu1} = 3{,}5, \quad when \quad f_{ck} \le 50 \quad MPa;$$
$$\varepsilon_{cu1} = 2{,}8 + 27[(98 - f_{cm})/100]^4, \quad when \quad f_{ck} \ge 50 \quad MPa. \quad (8)$$

2) subsequent calculation

$$\nu_{c1} = \sigma_{c1}/\sigma_{C1} = f_{cm}/E_{cm}\varepsilon_{c1}, \quad (9)$$

$\eta = \varepsilon_c / \varepsilon_{c1}$ and from formula (2a) – value σ_c.

The results' comparison of the calculation of concrete stresses σ_c, obtained by using formula (1a) of the regulation [1] and obtained by using formula (2a) of the method proposed in this paper, is illustrated in Fig. 3 and in the table. The obtained results of calculation using both methods for concrete stresses in strains ε_c interval from zero to ε_{cu1} are very close and their difference rarely reaches 5 %. It should be noted that the topicality of this problem is proved also by the research works of other researchers [8, 9].

Conclusion

Formula (2a), offered by the authors of this paper, can be used for the calculation of normal stresses of concrete of reinforced concrete members made of heavy and fine-grained concrete strength classes C 25/30–90/105 in strains ε_c interval from zero to ε_{cu1} instead of the formula of the regulation [1] (in this paper marked by number (1)). It is reasonable to do this in cases when there is a need to integrate the non-linear stress-strain diagram.

Fig.3. Graphs of relative values $\beta = \sigma_c / f_{cm}$ of stresses σ_c:
------ $\beta_{STR} = \sigma_c / f_{cm}$, when σ_c are calculated by formula (1a);
—— $\beta_{ZI} = \sigma_c / f_{cm}$, when σ_c are calculated by formula (2a)

The comparison of results of calculation of concrete stresses σ_c obtained by the offered and the regulation's [1] methods

Concrete class	Calculation method	$\eta = \varepsilon_c / \varepsilon_{c1}$																				
		0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1,0	1,1	1,2	1,3	1,4	1,5	1,6	1,7	1,8	1,9	2,0
C12/15	β_{ZI}*10³	0	242	442	604	732	830	901	950	980	996	1000	997	991	985	*983*						
	β_{STR}*10³	0	239	433	589	713	811	884	938	974	994	1000	994	977	951	*916*						
	β_{ZI}/β_{STR}, |%|	0	1,3	2,1	2,5	2,7	2,3	1,9	1,3	0,6	0,2	0,0	0,3	1,4	3,6	*7,3*						
C16/20	β_{ZI}*10³	0	227	419	577	706	807	884	939	975	994	1000	995	982	964	943	*922*					
	β_{STR}*10³	0	226	414	569	696	797	875	932	971	993	1000	993	974	944	903	*852*					
	β_{ZI}/β_{STR}, |%|	0	0,4	1,2	1,4	1,4	1,3	1,0	0,8	0,4	0,1	0,0	0,2	0,8	2,1	4,4	*8,2*					
C20/25	β_{ZI}*10³	0	216	400	556	685	789	870	930	970	993	1000	994	975	947	910	868	*821*				
	β_{STR}*10³	0	215	398	552	680	784	865	926	968	992	1000	993	971	936	889	830	*761*				
	β_{ZI}/β_{STR}, |%|	0	0,0	0,5	0,7	0,7	0,6	0,6	0,6	0,2	0,1	0,0	0,1	0,4	1,2	2,4	4,6	*7,9*				
C25/30	β_{ZI}*10³	0	204	382	535	665	771	856	921	966	992	1000	992	968	930	878	814	739	***653***			
	β_{STR}*10³	0	204	381	534	663	770	855	920	965	991	1000	992	967	926	871	801	717	***620***			
	β_{ZI}/β_{STR}, |%|	0	0,0	0,3	0,4	0,3	0,1	0,1	0,1	0,1	0,1	0,0	0,0	0,1	0,4	0,8	1,6	3,1	***5,3***			
C30/37	β_{ZI}*10³	0	194	367	518	648	757	845	914	962	991	1000	991	963	916	852	771	**672**	456	423	274	*110*
	β_{STR}*10³	0	194	367	518	648	757	845	913	962	991	1000	991	963	916	851	769	**669**	552	418	267	*099*
	β_{ZI}/β_{STR}, |%|	0	0,0	0,0	0,0	0,0	0,0	0,0	0,1	0,0	0,0	0,0	0,0	0,0	0,0	0,1	0,3	**0,4**	0,7	1,2	2,6	*11,1*
C40/50	β_{ZI}*10³	0	180	344	492	621	734	828	902	956	989	1000	989	954	896	812	**703**	569	*407*			
	β_{STR}*10³	0	180	344	491	621	733	827	901	956	989	1000	988	953	893	806	**693**	551	*379*			
	β_{ZI}/β_{STR}, |%|	0	0,0	0,0	0,2	0,0	0,1	0,1	0,1	0,0	0,0	0,0	0,1	0,1	0,3	0,7	**1,4**	3,3	*7,4*			
C50/60	β_{ZI}*10³	0	169	327	472	603	718	815	894	952	988	1000	987	948	880	**782**	*653*					
	β_{STR}*10³	0	169	325	469	599	713	811	890	950	987	1000	986	942	865	**750**	*593*					
	β_{ZI}/β_{STR}, |%|	0	0,0	0,6	0,6	0,7	0,7	0,5	0,2	0,2	0,1	0,0	0,1	0,6	1,7	**4,3**	*10,1*					
C55/67	β_{ZI}*10³	0	165	320	464	595	711	810	890	950	987	1000	987	945	**873**	*770*						
	β_{STR}*10³	0	164	317	459	589	704	803	885	947	986	1000	985	936	**848**	*715*						
	β_{ZI}/β_{STR}, |%|	0	0,6	0,9	1,1	1,0	1,0	0,9	0,6	0,3	0,1	0,0	0,2	1,0	**2,9**	*7,7*						
C60/75	β_{ZI}*10³	0	161	314	457	588	705	805	887	948	987	1000	986	**943**	868	*759*						
	β_{STR}*10³	0	160	310	450	579	695	796	879	944	985	1000	983	**929**	830	*675*						
	β_{ZI}/β_{STR}, |%|	0	0,6	1,3	1,6	1,6	1,5	1,1	0,9	0,4	0,2	0,0	0,3	**1,5**	4,6	*12,4*						
C70/85	β_{ZI}*10³	0	154	303	444	576	694	797	882	946	986	1000	**985**	939	*858*							
	β_{STR}*10³	0	152	297	434	562	678	781	869	938	983	1000	**980**	914	*785*							
	β_{ZI}/β_{STR}, |%|	0	1,3	2,0	2,3	2,5	2,4	2,0	1,5	0,9	0,3	0,0	**0,5**	2,7	*9,3*							
C80/95	β_{ZI}*10³	0	148	294	434	565	685	790	877	943	985	**1000**	984	935	*849*							
	β_{STR}*10³	0	146	286	420	546	662	768	859	932	981	**1000**	977	894	*723*							
	β_{ZI}/β_{STR}, |%|	0	1,4	2,8	3,3	3,5	3,5	2,9	2,1	1,2	0,4	**0,0**	0,7	4,6	*17,4*							
C90/105	β_{ZI}*10³	0	143	286	425	557	678	784	874	941	985	**1000**	984	*932*								
	β_{STR}*10³	0	140	276	407	531	648	755	848	925	979	**1000**	972	*869*								
	β_{ZI}/β_{STR}, |%|	0	2,1	3,6	4,4	4,9	4,6	3,8	3,1	1,7	0,6	**0,0**	1,2	*7,2*								

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S a n t r a u k a

LENGVAI INTEGRUOJAMOS BETONO ĮTEMPIŲ-DEFORMACIJŲ PRIKLAUSOMYBĖS SUPAPRASTINTAS VARIANTAS

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Kuriant ir tyrinėjant gelžbetonines konstrukcijas dažnai tenka apskaičiuoti įtempių-deformacijų būvį statmenuose elemento ašiai (normaliniuose) pjūviuose įvairiose apkrovimo stadijose: iki plyšimo, plyšių susidarymo metu, konstrukcijų su plyšiais eksploatavimo ir irimo stadijose.

Kadangi šiuo metu skaičiavimuose yra naudojamos labai netikslios betono įtempių-deformacijų tarpusavio priklausomybės diagramos, tai nėra galimybės apskaičiuoti plyšių aukštį ir betono tempiamos zonos virš jų aukštį. Itin apytiksliai nustatomas betono gniuždomosios zonos aukštis. Neįmanoma apskaičiuoti minėtų parametrų realių reikšmių plyšimo ir irimo stadijose ir prieš šias stadijas. Be to, negalima nustatyti šių parametrų nekarpytų sijų plastiniuose šarnyruose. Nėra metodo gelžbetoninių elementų įtempių-deformacijų būviui apskaičiuoti pagal išmatuotus normalinių plyšių parametrus (plyšių plotį ir aukštį bei atstumą tarp jų) arti irimo stadijos.

Naudojant realesnes kreivalinijines betono įtempių-deformacijų tarpusavio priklausomybės diagramas, daug minėtų uždavinių galima išspręsti [4–7] – galima nagrinėti apkrovų kitimą nuo nulio iki elemento irimo apkrovų, t.y. bet kokią apkrovimo stadiją nuo pradžios iki elemento suirimo, net ir elemento stiprumo mažėjimo stadiją, pavyzdžiui, plastiniuose šarnyruose. Kreivalinijinė diagrama ir atitinkama betono įtempių-deformacijų priklausomybės formulė pateikta reglamente [1]. Tačiau jos panaudojimą apsunkina integravimo problemos.

Šio straipsnio autoriai siūlo originalią paprastą formulę gelžbetoninių elementų, pagamintų iš sunkiojo ir smulkiagrūdžio C25/30–C90/105 stiprio klasių betono, normaliniams betono įtempiams apskaičiuoti. Skaičiavimo rezultatai, gauti pagal reglamento [1] formulę (1a) ir autorių pasiūlytą formulę (2a) yra artimi, skirtumas retai siekia 5%. Formulę (2a) tikslinga naudoti, kai reikia suintegruoti kreivalinijinę betono įtempių-deformacijų diagramą [4].

Temos aktualumą parodo ir kitų autorių mokslinio tyrimo darbai [8–10].

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